The Counterpoint Game: Rules, Constraints and Computational Spaces

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ABSTRACT

We envision first species counterpoint as a fundamental step in defining rules in a game-theoretical approach to algorithmic composition. While previous literature focuses on producing counterpoints that are aesthetically pleasing, our work formalizes the space where *cantus firmi* and counterpoints can be considered valid. A set of common rules is extracted from a comparative analysis of treatises from the 18th to the 20th century, and a generative algorithm is implemented to explore the validity space. The results highlight the existence of a subset of *cantus firmi* that do not admit valid counterpoint. Human experts in a qualitative assessment perform similarly to our validation algorithm; at the same time, the systematic omission of single and pairs of rules does not eliminate such a subset. It follows either that unwritten musical knowledge must be rendered explicit to modify the proposed rule set, or that, in general, admitting valid counterpoint is not a necessary property of *cantus firmus*.

1. INTRODUCTION

This paper questions the existence of a deterministic validity space for counterpoint. This section summarizes the evolution of counterpoint in music history and computer-aided composition, to clarify the scope of our research.

1.1 Counterpoint in Music History

The art of counterpoint was born when, at the beginning of the 13th century, composers started to build new lines upon pre-existing ones, opposing a new note (in Latin *punctum*) against each note of the original line (*contra punctum*). The line that originates the composition, called *cantus firmus*, synthesizes melodic organization in its simplest form: *"Harmonic implication, rhythmic profile and motivic design are all eliminated in order to allow the utmost concentration upon purely linear factors."* (Salzer and Schachter [1]). In the tradition, the *cantus firmus* is a short excerpt from the huge melodic repertoire of the Gregorian plain chants, and forms the basis for more elaborate contrapuntal compositions (see Figure 1 for an example).
The technique of counterpoint evolved through the work of early Renaissance composers, like Guillaume Dufay, Josquin Desprez and Giovanni da Palestrina, who in some cases wrote dozens of new compositions over the same *cantus firmus* (one famous case is the theme *"L'Homme Armé"*).
In 1725 the Austrian composer and music theorist Johann Joseph Fux published the pedagogical work "Gradus Ad Parnassum" [2], where he emphasized the need for a method to gradually introduce the students to master the difficult art of counterpoint. He organized his treatise in different "species counterpoint" exercises of growing difficulty.

1.2 Counterpoint in Automated Composition

The automatic generation of counterpoint is a classical problem in algorithmic composition. Literature approaches can be divided in two broad categories.
The first category corresponds to the historically oldest practice, constraint logic programming (see [3, 4, 5]). In this method the programmer explicitly enters rules as the knowledge base of the expert system, typically selecting them from a single authoritative source (as in [4, 5]). However, textbook rules usually prove to be insufficient, and heuristics become necessary to obtain the desired aesthetic results, at the expense of the generality of the model (e.g. [3]).
Approaches in the second category infer templates for composition directly from pre-existing data, disregarding *a priori* information. Two broad families of algorithms are employed: algorithms based on statistical emergence (such as [6, 7, 8, 9]), and algorithms for metaheuristic optimization (such as [10, 11, 12, 13]). Nonetheless, stochastic and machine learning systems, while leading to musically pleasing results, provide little to no insight into the internals of the musical language (e.g. [7]).
All previous work aims at composing counterpoints over a human-written *cantus firmus*, so that the result is musically pleasing. They question neither the validity nor the completeness of the underlying generative model.

1.3 Our contribution

Our work extracts from the textbooks a set of common rules that appear *necessary* (although potentially not sufficient) for correct first species counterpoint in every style and every historical period – and examines the characteristics of the resulting set of "valid" counterpoints.

We begin in Section 2 by extracting baseline indications from a comparative analysis of several treatises and textbooks spanning a wide range of styles and historical periods. Making some simplifying assumptions that allow

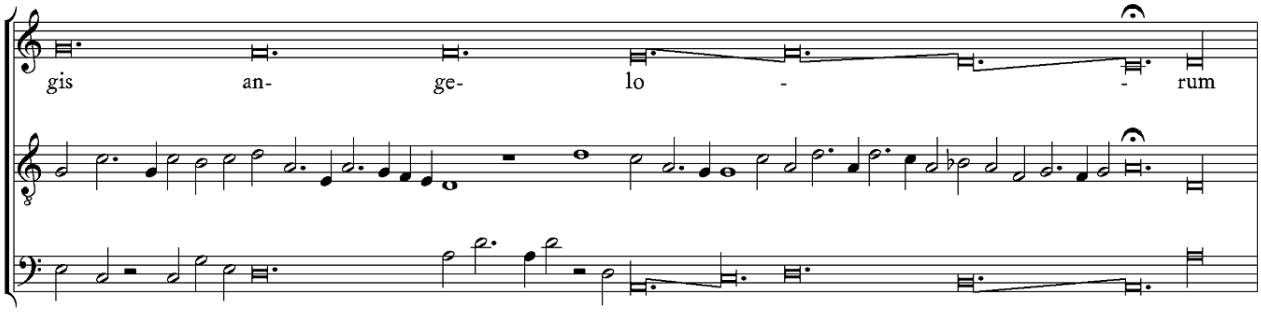

Figure 1. Ave Regina Caelorum, Anonymous XV century, Trento Cathedral, Italy (transcribed by Renato Calcaterra).

us to focus without loss of generality on the fundamental aspects of the prescriptions, we distill the various examples and guidelines into a set of deterministic, formal rules. Some of these rules apply only to *cantus firmus* (from now on CF), some only to counterpoint (from now on CP), and some to both; note that previous work never explicitly formalized CF. Violating at least one rule means a CF/CP runs against a precept spanning *all* styles and periods; we call these CFs/CPs *invalid*, as opposed to *valid* CFs/CPs that satisfy all rules. Obviously, a valid CF/CP may still run against the precepts of a *specific* style/period, have aesthetic issues, etc.

In Section 3, we design an algorithm to generate all valid CFs and CPs. We then analyse quantitatively and qualitatively the results; in particular, we examine the correlation between musicality of a given CF and the number of CPs it admits.

In Section 4, we verify the rules and evaluate their function: a comparison of human expert assessment and automated analysis corroborates our findings.

Finally, in Section 5 we summarize our results, examine their significance, and look at some possible avenues of future research.

2. CANTUS FIRMUS AND FIRST SPECIES COUNTERPOINT

In this section we determine the rules for validating CFs and CPs. We first describe the assumptions under which the rules are formulated (Subsection 2.1). We then justify our choice of sources, and explain our framework for rule formalization (Subsection 2.2). Subsections 2.3, 2.4 and 2.5 present the rules that affect respectively both musical lines, only CFs, and only CPs, discussing the most critical or controversial rules.

2.1 Assumptions

To better analyse the space of possible outcomes, we minimize the complexity of the composition of CF and CP. We employ only the Ionian mode, whose pitches correspond to the C major scale, as we want to target general voice leading problems rather than stylistic peculiarities, while we try to encompass both modal and tonal facets whenever possible. Furthermore we limit our production to a two part, first species counterpoint, where notes of the same duration are opposed one against the other, as defined in [2]. We position the CF in the lowest voice, and the CP in the highest: we use the contiguous ranges of tenor and alto which, according to the registers of the human voice, pertain to the note ranges B2-G4 and G3-C5 respectively.

2.2 Rule Formalization

Formalizing rules for generating CFs and CPs is a remarkably difficult task. Musicians have a long tradition in teaching, theoretical research, and treatise writing. Nevertheless, the goal of a music teacher is not to give his students a set of absolute rules, but rather to shape their musicality and stylistic awareness. Therefore, the language used in textbooks is often contradictory, unclear and indirect, to express tradition as well as musical intuition. Similarly, most information is conveyed by teachers to their students through examples and informal guidelines rather than through strictly codified rules.

To derive quantitative rules that define when CFs and CPs are valid, we first made a selection of relevant sources, covering differences in style as well as a wide historical period. Fux's treatise [2] is considered a classical book, referenced by many important composers, and celebrated as the first in the modern era to organize counterpoint studies in a rational way. Jeppesen's book [14] represents one of the main contributions in the direction of a philological study of Palestrina's counterpoint. Modal counterpoint is, further, the main focus of the more recent Zanolini and Dionisi's book [15]. On the opposite side, from a stylistic perspective, stand Schönberg's [16] and Salzer and Schachter's works [1], both devoted to a tonal counterpoint style.

After selecting the sources, we distilled and compared the existing rules. As mentioned above, many of the formulations were ambiguous or incomplete, and additional, unstated rules are implied by the examples. We therefore rephrased and integrated the rules, categorizing them into four classes, according to the level of agreement of the different sources on each rule. In the case of absolute rules (AR), i.e. when all authors agree, we included the rule without conditions; in the case of majority rules (MR), which occur when there is a partial disagreement between different sources, we chose the rule adopted by the ma-

AR, absolute rules	all the authors agree (some may not express judgement).
MR, majority rules	not all the authors share the same opinion.
UR, undefined rules	the rule is not clearly formulated and requires interpretation.
IR, implicit rules	the rule implicitly operates in the literature.

Table 1. Rule categorization.

jority of the authors; in the case of undefined rules (UR), whose interpretation is not unique, and implicit rules (IM), which are not expressed yet implicitly operate in the literature, we analysed examples in the sources to infer the correct formulation of the rule (see Table 1).
The following subsections illustrate "general" rules (that affect both CFs and CPs), *cantus firmus* rules, and counterpoint rules; furthermore, they discuss the endorsement of URs, IRs, and particularly crucial rules.

2.3 General Rules

General rules (Table 2) apply to both CFs and CPs. Out of nine rules, we individuated two AR, three IR and four MR: Rule G1 is a MR because there is no agreement among

G1. Only horizontal seconds, major and minor thirds, perfect fourths, fifths and octaves, and minor ascending sixths are allowed.	**MR**
G2. The 7th degree must resolve to the tonic, or it can descend stepwise if the preceding note is the tonic.	**IR**
G3. Do not reach a 7th degree by a skip larger than a third.	**MR**
G4. Successions of three notes cannot form an arpeggio.	**AR**
G5. If there are two horizontal intervals of equal distance and discordant, the next interval must differ.	**IR**
G6. Compound motion of any kind in one direction is allowed for no more than a sixth.	**IR**
G7. Compound movements which outline a tritone are forbidden.	**MR**
G8. Do not make stepwise concordant movements longer than a fifth.	**MR**
G9. A voice must not exceed the range of a tenth.	**AR**

Table 2. General rules.

authors: three of them (Fux, Jeppesen and Dionisi) allow only ascending minor sixths, Schönberg forbids all horizontal sixths, while Salzer and Schachter permit all of them. We chose to conform to the majority. Note that, in this case, even authors belonging to the same stylistic group disagree (tonal counterpoint, Schönberg vs. Salzer and Schachter).
Rule G2 is an IR: unexpectedly, no one of the authors mentions the rule outside the context of ending formulas. In tonal style, it is crucial to resolve the dissonant 7th degree ascending to the tonic, as this movement reinforces the sense of tonality. Tonal style authors might have given the rule for granted, as it is a direct consequence of classical harmony. In modal style the 7th degree does not always correspond to a leading tone: at the end of the piece, however, it can be artificially raised of a semitone to match its conclusive function. This justifies why modal style authors do not enforce the rule in the general case.
Excluding the rule would be incorrect from a tonal point of view; on the other hand, the Ionian mode contains the leading tone, and it can be considered a hybrid ground between a modal and tonal scale, as it corresponds to the C major scale - therefore, given our assumptions, we included the rule. After having analysed several examples, we also allowed the case where the 7th degree descends stepwise from the tonic (see Figure 2).

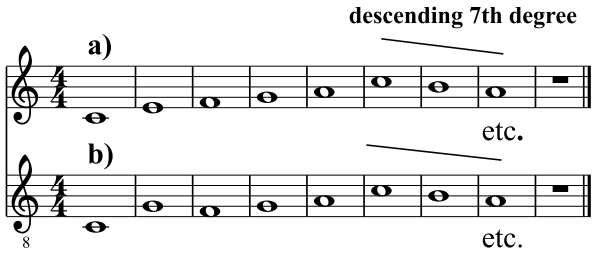

Figure 2. Salzer and Schachter [1], Chapter 1, selected examples.

Rule G5 is another case of IR. Many observations against redundancy, similar sequences and other forms of repetition can be found in all five textbooks we analysed. However, the supporting examples were extremely variable and context-dependant, and the prescriptions imprecise and subject to personal taste. Therefore, our rule vetoes only the most extreme form of repetition, the trill (see Figure 3).

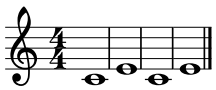

Figure 3. Example of trill.

Rule G6 (IR) is crucial for voice leading control. All treatises condemn excessive motion in one direction, yet they provide a number of partial constraints like:
a) avoid multiple skips in the same direction.
b) avoid compound movements which outline dissonant intervals.
c) leaps of ascending minor sixth or ascending/descending

octave must be recovered from.
Referring to Figure 4, constraint *"a"* can prevent situations like Ex.1 and 2, but not Ex.3.

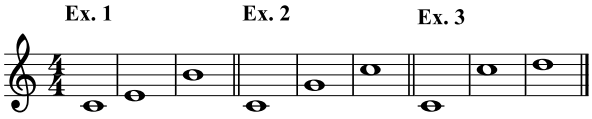

Figure 4. Examples of broad concordant movements.

Constraint *"b"* covers Ex.1 and 3 but not Ex.2, while constraint *"c"* covers only Ex.3. Our rule covers all the previous examples; moreover, it compromises with the rest of the constraints present in literature, following their general common principle.

2.4 Cantus Firmus Rules

Many counterpoint books present collections of historical *cantus firmi* to be used in students' exercises. Even if the practice of composing CFs is not completely disregarded, the treatises lack specific rules or indications. Nevertheless, we report four rules (Table 3): two AR, one MR and one UR.

CF1. The CF begins and ends on the 1st degree of the mode.	**AR**
CF2. The next-to-the-last note in the CF must either be a second or a seventh.	**MR**
CF3. In the CF a note cannot be immediately repeated.	**AR**
CF4. The CF must have a unique culmination (climax).	**UR**

Table 3. Cantus Firmus rules.

Rule CF2 is directly connected to its complementary rule CP3, because both define ending formulas for CFs and CPs. It is a MR as the authors largely agree, but admit variations which mostly depend on their referenced style - therefore we chose a more general formulation.
Rule CF3, shared by our references, derives from the way musicians adapted Gregorian lines into CFs. In fact, traditionally, Gregorian lines feature several repetitions. However, such repetitions used to be merged, in order not to interfere with the necessary rhythmic quality of the contrapuntal CF.
Rule CF4 was formulated in Salzer and Schachter as *"... Each cantus firmus must contain a climax ... The climax should not be repeated."*. All sources agree that there should be only one climax, save for exceptional cases when there should be "as few as possible". As we want all our rules to be deterministic, we settled for exactly one climax.

2.5 Counterpoint Rules

Counterpoint rules mainly address the vertical aspects of the composition. The only horizontal line prescriptions are Rule CP3, which allows oblique motion, and Rule CP11, which limits the use of excessive repetition in the CP . Out of eleven rules we have four AR, six MR and one UR, as seen in Table 4.

CP1. The beginning vertical intervals must be only positive perfect consonances, the ending positive octave or unison.	**MR**
CP2. The vertical intervals allowed are: minor/major thirds, sixths and tenths, perfect unisons, fifths, octaves.	**AR**
CP3. Do not repeat more than two consecutive whole notes.	**MR**
CP4. The next-to-last measure (CF and CP) must contain both the second degree and the leading tone.	**MR**
CP5. The unison can be used only at the beginning and at the end.	**AR**
CP6. In similar motion a note cannot cross the preceding note of the other voice.	**AR**
CP7. Any chain of two vertical fifths or octaves is forbidden.	**AR**
CP8. No perfect intervals can be taken by similar motion.	**MR**
CP9. No simultaneous skips in the same direction if both are greater than a fourth.	**UR**
CP10. No more than three thirds, sixths and tenths in a row.	**MR**
CP11. Do not use repetition more than twice.	**MR**

Table 4. Counterpoint rules.

Rule CP9 (UR) deals with simultaneous skips in both voices. The original rule by Salzer and Schachter was: *"... Whenever possible avoid simultaneous leaps, especially leaps in the same direction involving melodic intervals larger than a fourth."*. In Figure 5 we present different cases of simultaneous skips.

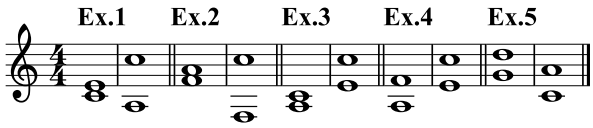

Figure 5. Five examples of simultaneous skips.

Salzer and Schachter's formulation is ambiguous in respect to all the examples in Figure 5, but not to Ex.3 and 4, where both voices make a skip in the same direction and larger than a fourth. How should we deal with Ex.1 and 2, where only one voice skips for more than a fourth? And what about Ex.5, where the skips are in the same direction? Most examples in the literature treat Ex.1, 2, and 5 as cor-

rect: this observation led to our formulation of the rule. Summarizing, the space of valid CFs and CPs is defined by a total of 24 rules. 21% of them are implicit or undefined rules. Only 33% of the rules are universally accepted.

3. CANTUS FIRMUS AND COUNTERPOINT SPACE ANALYSIS

The rules in the previous section classify every possible CF and CP as valid or invalid. This section illustrates an algorithm to generate all valid CFs and CPs, and analyses the results, examining the relationship between the musicality of each CF and the number of CPs it admits.

3.1 Generative Algorithm

We implemented software to explore the space of CFs and CPs that are valid - i.e., the set of the CPs and CFs that adhere to the rules in Section 2. The algorithm follows a simple generative pattern, which is applied to both CFs and CPs (taking care of the the necessary differences). Each iteration of the main loop of the algorithm appends a new note to the current voice line. In the case of the generation of a CF, at each step the algorithm produces the set of potential next notes, according to voice range, scale structure and current voice line prefix. This set is then pruned according to stationary rules (which are only conditioned by the position within the voice line), and horizontal rules (which depend on a suffix of the prefix of the voice line). The remaining set contains all valid notes that can be appended to the current voice line prefix.
In the case of a CP, the generation of the n^{th} note in the voice line also depends on a prefix of the corresponding CF, as the set of potential next notes undergoes the further filter of vertical and combined horizontal-vertical rules.
The algorithm uses the patterns above to build a suffix tree of all valid CFs (respectively, all valid CPs given a CF) where the root of the tree is a placeholder, each other node contains a note, and the sequence of the notes encountered in the path from the root to a node represent a valid voice line prefix.
Note that the extensive generation of all CFs and CPs for a given voice line length is computationally demanding: the cardinality of the output grows exponentially with voice line length, and a considerable number of voice line prefixes that do not allow for continuation are pruned during the generation process.

3.2 Analysis of Results

We generated all valid CFs, using a voice line length of 8 notes, as most literature does (e.g. [1, 16]). The output consisted of 4587 valid CFs. For each of the CFs, we then generated all valid CPs. We investigated the correlation between the aesthetic features of the CFs, and the number of admitted CPs - i.e., if a CF that is musically pleasing admits more, fewer, or as many CPs than a CF that is not. Therefore, we focused on a qualitatively significant subset of all valid CFs, according to the categories reported in Table 5. The first four categories are based on the number of CPs that the CFs admits. The remaining three categories refer to features of the CFs considered important in human judgement.

BAD	CFs that do not admit any counterpoint.
POP	CFs that allow the most frequently allowed number of counterpoints.
MID	CFs that allow the median number of counterpoints.
CTPP	the CF that allows the most counterpoints.
OCT	CFs that feature two consecutive octave intervals.
FOURTH	CFs that feature the range of a fourth.
TENTH	CFs that feature the range of a tenth.

Table 5. Categories of CFs.

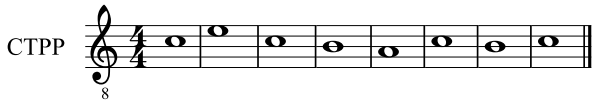

Figure 6. The CF which admits the maximum number of CPs over 8 notes.

The CF that allows the maximum number of possible counterpoints, in Figure 6, admits 273 CPs. The CF appears ordinary; though one could say that its insistence on note C3 (repeated four times) detracts from its voice leading.
The group of CFs with the highest number of CPs (POP), and the group with the median number (MID) feature CFs that could be qualitatively judged as either poor, average or good, as it can be seen in the examples in Figures 7 and 8.
To our surprise, we discovered the existence of 163 CFs which do not admit any valid CP. We labelled such CFs as "uncounterpointables". Two examples of uncounterpointables are presented in Figure 9. The first appears clearly faulty. It sports three octave skips – two of which in direct succession, thus leaving from and ending on the same note: such behaviour causes a surplus of movement that brings a sense of instability to the CF. Per contra, on a qualitative evaluation of the second uncounterpointable, nothing wrong is immediately apparent. It features correct voice leading, with good alternation of skips and stepwise movements; it has no repetitions nor abrupt large skips - it could

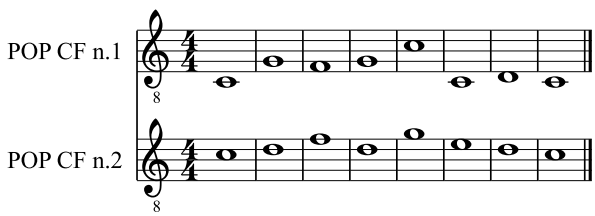

Figure 7. Two examples of CFs from the group POP.

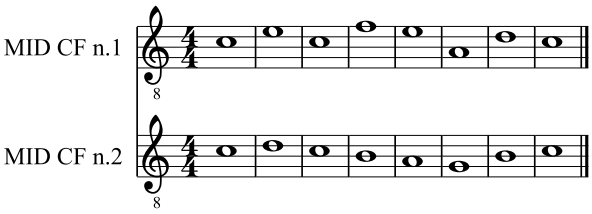

Figure 8. Two examples of CFs from the group MID.

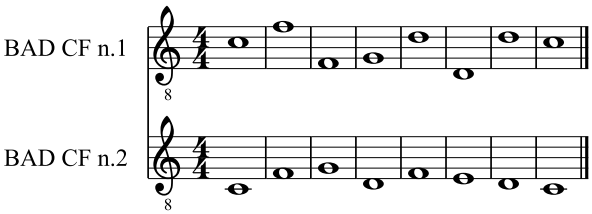

Figure 9. Two examples of CFs from the group BAD.

easily be considered a good musical example.
In brief, the existence of uncounterpointable CFs is not completely consistent with musical intuition. However, the number of CPs a given CF admits appears independent both from the quality of the voice leading of the CF, and from the overall "quality" of the CF. Notably, there exist uncounterpointable CFs that are also musically pleasing, while apparently very poor CFs can admit many valid counterpoint solutions, like the example in Figure 10.

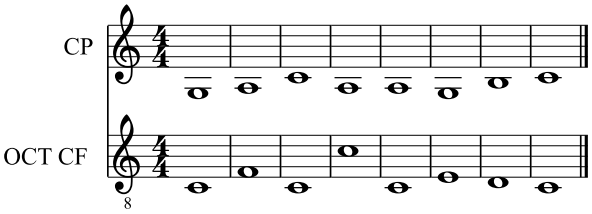

Figure 10. An example of a CP with good voice leading, over a CF, from the group OCT, with poor voice leading.

4. THE ROLE OF RULES

This section verifies the rules and the results presented in the previous sections. Specifically, it was crucial to understand if the origin of CFs that are not musical, and of uncounterpointable CFs, was due to a rule set that was too strict, thus not allowing valid CPs, or too lax, thus generating invalid CFs. We tested these hypotheses against examples from the literature (Subsection 4.1) and human experts (Subsections 4.2 and 4.3), and we systematically probed the rule set to better understand the role of each rule (Subsection 4.4).

4.1 Examples from Literature

We validated CFs taken from the literature; in particular, we extracted 6 CFs composed by Schenker and Jeppesen from Salzer & Schachter's manual, pp. 10-11 [1], and 6 CFs from Fux's treatise [2]. While all of them admitted counterpoint, some of them contravened one or more rules. In particular, Schenker disobeyed Rule G7 in one example, and Fux Rules G2, G4, G6 and G8. In respect to our research on academic literature, only the case of Fux breaking G2 could be considered a false negative (due to the formulation of the rules in Ionian mode, where the seventh degree is the leading tone, while the example was in Phrygian). The remaining rules were contravened in such a way that they would be considered invalid by the majority of the authors. We were particularly surprised by the repeated violation of G4 (avoid arpeggios), which is an absolute rule.

4.2 Expert Insight - Questionnaire

We compared the assessment of experts with automated validation results. Twelve composers and counterpoint teachers of Italian Conservatories were asked through an anonymous online form to comment on the validity of, and eventually correct, several CFs and CPs. More specifically, we asked them, disregarding aesthetic considerations, to point out formal errors in proposed CFs and CPs, and to compose exemplary CFs as well as CPs over given CFs. All CFs were selected from random samples from the categories in Table 5.
The evaluation of the CFs is summarized in Table 6.

CF group	Validity	Reported errors
MID	82%	two consecutive large skips (fifth, fourth).
BAD (good)	60%	two consecutive large skips (fifth, fourth).
CTPP	80%	wrong final formula (B3, C4); too many iterations of the same note.
TENTH	64%	presence of an octave skip; range too large (tenth).
FOURTH	100%	-
BAD (poor)	18%	too many skips.
OCT	18%	too many skips involving the same note.

Table 6. Expert evaluation of CFs. The first CF from the group BAD has good voice leading, while the second has poor voice leading. Validity indicates the percentage of authors that consider the CF without mistakes.

The main criticism to the automatically-generated CFs is the entity and frequency of large skips: this is the most common - and often first - remark. However, skips of the same entity and with the same frequency have not been marked as undesirable in different CFs, which means that

the context of the skip affects its alleged validity. Note that even the most criticized CF did not obtain 100% refusals, while the CF in the group FOURTH obtained 100% acceptance. This seems to indicate that the reduced voice range is highly appreciated, possibly because it prevents large skips. However, a narrower range also implies an increase in note repetition, a feature highly criticized in other CFs by our respondents.

Among the chosen CFs we provided two from the group BAD, of which one featured poor voice leading, while the other presented a more pleasing voice leading: most interestingly, the former obtains only 18% acceptance, while the latter 60%. This might indicate the presence of aesthetic factors in user judgement, and, most importantly, it confirms uncounterpointability is uncorrelated with formal validity in the eye of the musician.

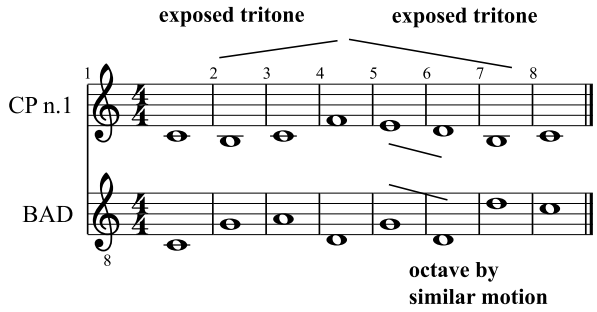

Figure 11. First counterpoint proposed for expert evaluation.

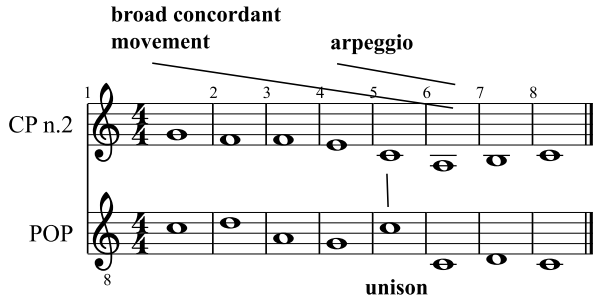

Figure 12. Second counterpoint proposed for expert evaluation.

We proposed four CPs for evaluation. 44.4% of the test takers corrected the octave vertical interval taken by similar motion in CP n.1 (Figure 11), while no one individuated the exposed tritones. In CP n.2 (Figure 12) 62.5% of the test takers corrected the unison vertical interval, only 11% marked as invalid the arpeggio, and no one spotted the broad concordant movement. The rule regarding exposed tritones admits in some textbooks an exception, when the tritone is resolved stepwise in the opposite direction, which is the case of the given example. The rule regarding arpeggios, in particular, had already been repeatedly violated in examples from the literature, as presented in Subsection 4.1.

Of the last two proposed CPs, which were automatically generated, one was mostly considered correct, while the other did not find corrections shared by a significant portion of the participants.

All user-composed CFs were considered valid by the automated system, with the exception of two of them containing an arpeggio. User-composed CPs were not taken into consideration due to technical reasons: in particular the non-intuitive mechanism for entering notes in the online form, and the possibly confusing use of the octave-lowered G-clef in the tenor part.

Summarizing, human experts appear to endorse the proposed rules, although a fraction of them considers admissible both outlined tritones when resolved stepwise and arpeggios.

4.3 Expert Insight - Interview

We interviewed individually three counterpoint teachers, to gain additional insight. During the interview we asked them to elaborate on the validity of the rules criticized in the previous tests, through open questions and examples. All interviewees stressed the existence of discrepancies between textbook precepts and compositional practice that are traditionally accepted. Two out of three condoned exposed tritones when resolved stepwise in the opposite direction. All three found the closing formula rules to be somewhat too strict, yet they did not agree on a modification proposal. Interviews shed light upon the validity of arpeggios: CFs were commonly excerpts of Gregorian chants which, preceding harmony theory, could outline arpeggios. However, even if forbidden by counterpoint compositional rules, arpeggios were often not removed in the adaptation of the chant as a CF, to maintain the characteristics of the original voice. Given the didactic function of CFs, these were often passed on through generations of teachers unconditionally.

4.4 Systematic Rule Exclusion

Finally, we systematically tested the exclusion of a single rule, and of pairs of rules, from the automated system. No single rule eliminates the presence of uncounterpointable CFs. Rules CP5 (unison), CP8 (perfect interval by similar motion), and G2 (resolution of the 7th degree) are by far the most selective rules: excluding each rule in turn, the ratio between uncounterpointable CFs and generated CFs is respectively 0.04%, 0.17%, 0.08%, using a voice length of 8 notes.

Rule G6, regarding broad concordant movements, does not eliminate uncounterpointable CFs even when paired with other rules. Modifying Rules G1 and G7 to allow tritones when resolved stepwise (both changing direction and continuing in the same direction) while simultaneously allowing arpeggios still generates uncounterpointable CFs for a voice length of 8 notes.

Excluding *pairs* of rules does not significantly reduce uncounterpointability. The only pair that eliminates uncounterpointable CFs for a voice length of 8 notes, and that agrees with experts' observations, is G2 (that prevents arpeggios in both voices) and G4 (resolution of the 7th degree).

However, increasing the length of the CF to 11 notes, new uncounterpointable CFs appear, even excluding these rules.

5. CONCLUSIONS AND FUTURE WORK

We extracted a set of common rules from notable treatises for different historical and stylistic contexts. As all treaties presented inconsistencies or omissions, the set of rules required integrations through a comparative analysis of the sources.

It appears that the number of counterpoints a CF admits is uncorrelated with its musicality, and that there exist uncounterpointable CFs - i.e., CFs that do not admit valid counterpoint. Considering these findings, we validated the rule set against composers and counterpoint teachers. Experts generally agreed on two main observations: outlined tritones that resolve stepwise could be considered valid, and arpeggios might appear (only) in the CF, mimicking Gregorian style. A systematic analysis of the rules nonetheless demonstrated that uncounterpointable CFs continue to exist even after the proposed modifications.

It is then natural to ask: *"is it possible to formalize a validity space for CFs and CPs using deterministic rules?"*. If the answer is no, the relative correctness of a counterpoint can only be seen as an optimization problem – where each violated rule determines a penalty, and the goal is to minimize the overall loss. If the answer is yes, one must face two possible alternatives. The first is that the presence of uncounterpointable CFs is an inconsistency, and the rule set must be modified. However, since human experts could not individuate a point of failure in the rules, and textbooks lacked general and unambiguous directives, currently unexpressed musical knowledge on the fundamentals of counterpoint needs to be made explicit. Alternatively, the presence of uncounterpointable CFs is an inherent, uncharted feature of the space of valid CFs. It would be of great interest to understand if such CFs can indeed be formally characterized.

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